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The Generalized Die Binomial Experiment

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Abstract

The theoretical idea of a six-sided die binomial experiment, in the theory of probability distributions is examined in this paper. This idea is extended and generalized in the postulate of an n-sided die binomial experiment in which there are n finite possible outcomes of such die toss, for m- trials with equal probabilities p and $1-p$ of success and failure, respectively, for each trial. Theoretical results of binomial distributions are extended and relationships with existing results are deduced. Potential applications abound in statistics, social sciences, engineering and physical sciences, pertaining to sampling and chance minimization.

Keywords: Binomial Experiment; Six-sided Die; Generalized Die.

1. Introduction

If an experiment can have only two possible outcomes, such an experiment is called a Bernoulli experiment according to [2, 3, 4]. Simple examples of Bernoulli experiments are games of chance (rolling a die, tossing a coin, etc.), quality inspection (e.g. count of the number of defectives), opinion polls (number of employees favoring certain schedule changes, etc.), medicine (e.g. number of patients recovered by a new medication), and so on. See [1, 5].

In a Bernoulli experiment the probability of one outcome usually denoted by p is called the probability of success, and the probability of the other denoted by $q=1-p$ is called the probability of failure [4].

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A random variable, say X, is usually used in representing the number of success in a fixed number m of identical and independent Bernoulli trials [3]. Such a random variable is also said to have a binomial distribution with the probability function given by:

$$b[x:m,p]=\begin{bmatrix} m \\ x \end{bmatrix} p^x q^{m-x}, \quad x=1,2,\ldots,m$$

This is truly a probability function since

$$\sum_{i=1}^{m} b[x:m,p]=[p+q]^m=1$$

This paper acknowledges the fact that extensive research has been done on Bernoulli/Binomial experiments with restrictions to the aspect of games of chance, and emphasis on the die-rolling experiment by prominent ancient, medieval and contemporary mathematicians and statisticians. However, in terms of the die-rolling experiment, much emphasis has been placed on six-sided die Binomial experiments without ramified considerations given to an awkward possibility of performing the same die-rolling experiment but, this time, with a constructed die which has more than six sides. Indeed, this idea of performing the binomial experiment with a constructed die having more than six sides may sound awkward and may even appear trivial but the developments from it are significant.

2. Definitions

2.1 Binomial Experiment

A binomial experiment is an experiment which satisfies the four conditions that: (1) there is a fixed number of trials (2) each trial is independent of the others (3) there are only two outcomes for each trial and (4) the probability of each outcome remains constant from trial to trial. A binomial experiment is also known as a Bernoulli trial [2].

2.2 Binomial Theorem

In probability theory and statistics, the binomial distribution is the discrete probability distribution of the number of successes in a sequence of m independent yes/no experiment, each of which yields success with probability p [1]. It can also be defined as the distribution of a number of successes in a given number of Bernoulli trials [5]. The common notation is $b[x:m,p]$, where x is the number of successes, m is the number of trials, p is the probability of success. Where:

$$b[x:m,p]=\begin{bmatrix} m \\ x \end{bmatrix} p^x q^{m-x}, \quad x=1,2,\ldots,m$$

2.3 Mean, variance, standard deviation, moment coefficient of skewness, moment coefficient of kurtosis and moment generating function

If a random variable X has a binomial distribution with parameters m and p then:

* mean, $\mu_x = mp$

* variance, $\sigma_x^2 = mpq$

* standard deviation, $\sigma_x = \sqrt{mpq}$

* moment coefficient of skewness, $\alpha_3 = \dfrac{q-p}{\sqrt{mpq}}$

* moment coefficient of kurtosis, $\alpha_4 = \dfrac{[1-6pq]}{mpq}$

* moment generating function, $m_x[t] = [pe^t + q]^m$

3. Six-sided die binomial experiment

The experiment of tossing a six-sided die follows a probability distribution that is binomial with known probabilities $p = 1/6$ and $q = 1 - p = 5/6$ of success and failures respectively; and with these probabilities it investigates the behavior arising from rolling the six-sided die for m number of times, using the resulting distribution function, and statistical measure of definition 3 formed by substituting for p and q in the original models. This it does as follows:

We know that the random variable X representing the number of success x in a fixed number m of identical and independent Bernoulli trials is said to have a binomial distribution with the probability function:

$$b[x : m,\ p] = \begin{bmatrix} m \\ x \end{bmatrix} p^x q^{m-x}$$

Since $p = 1/6$ and $q = 5/6$ we have that

This gives the distribution function for a binomial experiment involving rolling a six-sided die [normal die] for m number of times. Furthermore, substituting for the values of p and q into the mean, variance, standard deviation, moment coefficient of skewness, moment coefficient and moment generating function of the binomial random variable X respectively yields:

$$b[x : m,\ p] = \begin{bmatrix} m \\ x \end{bmatrix} \left[\frac{1}{6}\right]^{x} \left[\frac{5}{6}\right]^{m-x}$$
$$= \begin{bmatrix} m \\ x \end{bmatrix} \left[\frac{1}{6^{x}}\right] \left[\frac{5^{m-x}}{6^{m-x}}\right]$$
$$= \begin{bmatrix} m \\ x \end{bmatrix} \left[\frac{5^{m-x}}{6^{m}}\right]$$

* $\mu_x = \dfrac{m}{6}$

* $\sigma_x^2 = \dfrac{5m}{36}$

* $\sigma_x = \dfrac{\sqrt{5m}}{6}$

* $\alpha_3 = \dfrac{4}{\sqrt{5m}}$

* $\alpha_4 = \dfrac{6}{5m}$

* $m_x[t] = \left[\dfrac{(e^{t} + 5)}{6}\right]^{m}$

4. The generalization

This section extends and generalizes the ideas of the previous section. It postulates the possibility of remodeling and extending the experiment of tossing a six-sided die to that of tossing a constructed die with more than six sides having numerical inscriptions ranging from 1 to $\mathbf{n}$ [on each side], and tossed for $\mathbf{m}$ number of times. Based on this postulate, we have deduced that the expectation, variance, standard deviation, moment coefficient of skewness, moment coefficient of kurtosis, and moment generating function for this posited experiment respectively would be:

* $\mu_x = \dfrac{m}{n}$

* $\sigma_x^2 = \frac{[m(n-1)]}{n^2}$

* $\sigma_x = \sqrt{\frac{[m(n-1)]}{n^2}}$

* $\alpha_3 = \frac{n-2}{\sqrt{m(n-1)}}$

* $\alpha_4 = \frac{7-6n}{m[n-1]}$

* $m_x[t] = \left[\frac{(e^t + n - 1)}{n}\right]^m$.

Such an experiment is truly a binomial experiment since

* it has a fixed number of trials, m

* each trial is identical and independent of the others

* there are only two possible outcomes for each trial and

* the probability of success denoted by p is the same on every trial.

5. Theoretical implications

An immediate implication of our postulate is that the binomial experiment of tossing a die of six sides becomes a specific case of our generalization since theorems that hold for the specific case would hold for the generalization and vice versa under proper assumptions on stability of the die. More so, our generalization indicates that without knowledge of the probabilities of success or failure one can still predict the expectations, variance and moment generating function of X . As an example, let X be a random variable representing an even outcome. Then the probability of getting one (1) even outcome in five (5) trials is given as follows.

We notice that increasing the number of sides infinitely minimizes the chances of occurrence of an outcome in the die binomial experiment. This implies that, in general, as $n \to \infty$, $b[x:m,\ p] \to 0$. Hence,

This is because $\lim_{n\to\infty} n^m >> \lim_{n\to\infty} [n-1]^{m-x}$. Hence, $\lim_{n\to\infty} b[x:m,\ p] = 0$ slowly, and so does **6.**

$$\lim_{n\to\infty} b[x:m,\ p] = \lim_{n\to\infty}\begin{bmatrix} m \\ x \end{bmatrix}\left[\frac{[n-1]^{m-x}}{n^m}\right]$$
$$= \begin{bmatrix} m \\ x \end{bmatrix}\left[\frac{\lim_{n\to\infty}[n-1]^{m-x}}{\lim_{n\to\infty} n^m}\right]$$
$$= 0$$

Table 1: Probability of obtaining one even outcome in five trials

Number of sides	$f(x)$
6	$b[x:5, 1/6] = 0.3215$
8	$b[x:5, 1/8] = 0.2931$
10	$b[x:5, 1/10] = 0.2624$
12	$b[x:5, 1/12] = 0.2354$
16	$b[x:5, 1/16] = 0.1931$
20	$b[x:5, 1/20] = 0.1629$
100	$b[x:5, 1/100] = 0.0384$
1000	$b[x:5, 1/1000] = 0.0040$
1000000	$b[x:5, 1/1000000] = 0.00000263$

6. Potential applications

(a) Potential applications of this development abound in statistics. For instance, in carrying out simple random sampling on a population for the purpose of allocating attributes to different samples drawn from such a population, one could design a die with sides equal [in number] to the number of samples in the population and then toss [for a fixed number of trials] the die, bearing in mind that each side of the die has been randomly assigned to the different sample of the population. This is preferred to a case of flipping a coin several times for the same purpose.

Precisely, suppose the government intends to distribute k limited mosquito nets to $m\ (m > k)$ households in a given community within a Local Government Area of a State. This would ordinarily require a pattern of distribution with minimal bias so that each of the $k < m$ households will get a mosquito net with equal probability $1/k$. Most often, a die of six sides would be employed in sampling $n = k$ households to which will benefit from the process with equal probability of $1/6$. However, employing the idea of a $k < m$ sided die ensures the distribution is done in time with $1/k$ equal probabilities for each household. This development must not necessarily be done manually, but can be achieved using a computer program.

(b) Furthermore, minimizing the probability of occurrence of an event in a die experiment could be achieved by increasing the number of sides of the die in question.

7. Conclusion

The idea of a binomial experiment has been studied in the context of generalizing the binomial experiment of tossing a six-sided die to an n-sided die for a fixed number of times. The generalized results were obtained for the mean, variance, standard deviation, moment coefficient of skewness, moment coefficient of kurtosis, as well as moment generating function. These results show that the experiment with $n > 6$ sided die was truly a binomial experiment. Moreover, it was shown that as the number of sides in the die increases, the binomial probability approaches zero.

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